Special Relativity today.

The theory of special relativity is currently in a surprising state. I say the theory , because in practice it work's quite well ; the interpretation errors do not typically affect the outcome. However , these mistakes hinder a deeper understanding of relativity , and lead to the production of a rather superfluous literature.

We could summarise this situation in one sentence : does a time unit really exist ? Or in a more detailed manner : are the rates of modern clocks completely unsensitive to motion (within defined limits , although in the case of atomic clocks , these may be close to infinite). It is highly probable that most clockmakers would give a similar answer to that question , but the aim here is to get an answer theory could provide.

We will start from the beginning , and what better way to do so than with Albert Einstein ? Numerous others have theorised on the question of relativity , but it is him who laid the first postulate : « The laws by which the states of the physical systems undergo changes are not affected , whether changes of state be to the one or the other of two sytems of co-ordinates in uniform translatory motion ». This obviously implies that clocks must belong to said « physical systems » , so clock rhythms should stay constant during that type of motion. Unfortunately Einstein never explicitly stated so !

Instead , he considered the rates of moving clocks as seen by a non-moving observer. One of his main cited inspirations came from the railroad : the time difference between the train clock and the time on the train platform would grow from train station to train station. The most logical explanation would be that the clock on board of the train was running late[1] … .

Einstein has thus demonstrated (somewhat laboriously) the Lorentz transformation , without formulating any hypothesis on clocks working. This meant he didn't have to think as though clocks in motion were indeed running late. As a clever physicist once told me , Einstein's mind was working with theorical clocks rather than real clocks. I had never thought about it in that way ! Moreover , Lorentz and Poincaré , like Einstein , only worked using coordinates , without much care for which units they used , and thus neglecting to question how a real clock[2] works !

But in the meanwhile , Minkovski created the concept of spacetime ! That is a metric space , that can be defined in a completely abstract manner , without the usual need for « concrete » portrayal.

Let us define in a pseudo-orthonormal frame :

1/ $\quad Q(x,y) = x.y$

2/ $\quad (e_0)^2 = 1 \qquad (e_1)^2 = -1 \qquad e_0 . e_1 = 0$

thus defining also the metrical tensor.

We may add that for any other pseudo-orthonomal frame we can write the same relations , completed by :

3/ $\quad e_0 . e'_0 = e_1 . e'_1 = \gamma$

Of course both of these frames are strictly equivalent. We can imagine them moving with the relative velocity $\pm v$. And we see now that relation (3) is sufficient to ensure that each clock is seen running late from the other frame , and with the same γ … !

This is or course coherent with both clocks being unsensible to motion. It could not be otherwise !

Another thing to highlight regarding relation (3) is that it is sufficient to prove the simplified form of Lorentz transformation !

1. What Einstein had in his mind is not so clear. In the same note he introduced unknowingly what will later on be called the « clock postulate » by describing a sort of circular trajectory . One must suppose that he admitted de facto the constancy of the clock rhythm !

2. Einstein must have feelt guilty regarding his carelessness , as he needed to detail the nature (« Unruh Uhr ») of the clock traveling between the Equator and the poles.

Quite usefully , this tensorial demonstration could be completed by geometric algebra. We can even say should the units of time and space be ignored , then geometric algebra[3] would be considered wrong !

A small mathematical intermission.

I was lucky enough to first learn about relativity from a wonderful book by P.K.Raschewski , « Riemannsche Geometry und Tensoranalysis ». At least he hadn't forgotten his vectors ! How is it possible to justify a demonstration on Wikipedia when you cannot find the essential relation (3) in any book[4] , whether in English or in French , accepted by the moderators ? They would soon kick you out of the door !

Let's start again ! We are sending an astronaut towards a remote planet at speed v . He arrives there at time τ according to his clock (we should really be calling it a second-dial …). Lorentz transformations state that this time is « measured » – in fact , calculated – by a observator staying on Earth as t :

4/ $$t = \gamma\tau \qquad \text{with} \qquad \gamma = (1 - v^2)^{-1/2}$$

We are able to verify[5] that fact by noting that the vector $e_0\, t$ is the orthogonal projection of the vector $e_0'\, \tau$. Indeed we have :

5/ $$0 = (e_0\, t - e_0'\, \tau).e_0 = t - \gamma\tau$$

Reciprocity results from a vector $e_0\, t'$ such as :

6/ $$0 = (e_0\, t' - e_0'\, \tau).e_0' = \gamma t' - \tau$$

I suggest you to compare with the wikipedia articles , french and english , dealing with « dilatation du temps ».

Whether you believe me or not , I have been told that my calculations , although right , were useless to the scientific community since they were not published anywere … ! I am therefore supposed to be the only physicist in the world (!) to have mathematically expressed a well known fact : the simultaneity surface is orthogonal to time !

Really , this is a profound problem : beginners (and even a few amateurs believing to be advanced physicists) studying relativity often use this lack in theory to object to the existence of relativity. The answer to these objections is actually quite simple : well-made clocks are not influenced by motion ! The symmetry comes naturally from relation (3). But that is exactly what we do not tell them !

But why ??

Let us rewind just a little. Had Einstein been able to anticipate the existence of spacetime , and had he known how to use unit vectors , he could have written :

7/ $$e_0'\, \tau = e_0'\, \gamma^{-1}\, t = (e_0'\, \gamma^{-1})t = e_0'(\gamma^{-1}\, t)$$

He would therefore have been able to choose between both interpretations. I, personally have no doubts that it is the second explanation that best represents physical reality.The first option , favoured by numerous mathematicians and physicists , would be an « als ob » variant which was unfortunately retained as the main explanation. The effects on the theory are dire : the scalar measure of time is wrongly replaced by the fake rhythm of a (vectorial …) clock. The great and the minor scientists followed (prudentially …?) Einstein. Well , why is there a problem ? The math seem to be the same ?

I first discovered this issue when venturing around Wikipedia : surprisingly enough , a taboo seemed to have have wrapped around the comparison between clock rhythms. It is impossible nowadays to write that two identical clocks moving with some $\Delta v \neq 0$ could have the same rhythm. It was even said that this comparison was nonsensical , even in the case of atomic clocks.

3. One can construct special relativity by the unique formula $ab = a.b + a \wedge b$
4. In fact I have found that relation in only one modern book : Gourgoulhon Relativité Générale .
5. To demonstrate γ calculate ds^2

Physicists , whether well-recognized or newcomers , have done little to address this fake mystery , with the exception of Professor Lévy-Leblond[6]. Never do they state that clock rhythms do diverge , but nowhere do they state that clock rhythms do not diverge either , except for a few scatterbrains. It is therefore impossible to provide any source that would correct the misleading information found on Wikipedia. This should be very frustrating for students.

What seems clear is that professional teachers should review their explanations. Let's look at an example , in which we will put aside that units of time and space have been unified. Lorentz and Poincaré wondered about the reality of shortening of distances across a rocket. Nobody today believes that the platinum rod shrinks ! So why should we believe that a clock in motion actually runs late ! But these units have been unified[7][8],so no question remains !

Unfortunately for Wikipedia readers , adopting an interpretation closer to the physical reality would dismantle numerous false issues they may wonder about. Too bad !

A few additional details concerning the so called clock postulate.

We have to extend the principles of relativity to reference frames moving freely , that is with accelerations and whithout gravity , thus staying in special relativity.

The literature regarding these principles is extensive , and I was only able to sample a very small part of it.What comes out of my readings however is the loyalty to Einstein's interpretation of clocks. It is thus no wonder difficulties in theoretical interpretation exist !

Just like Professor Lévy-Leblond , I am rather keen on chrono-geometry , so I consider that the spacetime metric allows to approximate almost every trajectory by small linear segments. Thus we can write :

8/ $\gamma_n \, d\tau = dt$ with $\gamma_n = e_n \, . e_0 = (1 - v_n^2)^{-1/2}$ and $e_n^2 = e_0^2 = 1$

where γ is by v_n a function of t

9/ $\Delta\tau = \int d\tau = \int \gamma^{-1} dt$

This highlights the vectorial caracter of γ , which is sufficient to assure that the rates of the clocks stay constant. From this interpretation , we can state that the clock postulate is in fact a theorem.

One may wonder why well-known textbooks[9] define what is called « Wristwatch Time » , but nowhere in these books it is mentioned that the rhythm of this clock does not depend on motion. Why ? …

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6. Levy-Leblond Le paradox des jumeaux

7. I believe this unification would not have been possible without the existence of atomic clocks , which we consider to be almost infinitely precise.

8. Morover , it is quite simple to demonstrate that the proper time of a linear path is independent of the reference frame. It is therefore obvious that a clock's rhythm does not vary with motion.

9. Spacetime Physics E.F.Taylor J.A.Wheeler