Time dilation in Special Relativity.

A new interpretation ... ?

When the young Einstein published his article on what was later called « Special Relativity » , he gave not only the physical and mathematical proofs of what he asserted , but also he gave the first interpretation of it. Then he knew nothing of what was later called the « Minkowski Vector Space ».

More than 100 years later that physical speciality is still poorly treated in the mainstream of Physics. Indeed cosmologists or quantum physicists work with General Relativity and tensorial methods. They dont loose much time with original Einstein interpretations piously maintained by his followers.

So in every book for undergraduate students you find the traditional texts for « time dilation » and « length contraction » , or « twin paradox » .... , without fully exploiting the Minkowski space. Moreover I am sure that a vast majority of readers think that it is not possible to define a unit of time in SR. Nobody thinks that a test unit of lenght will actually shrink if transported in a rocket, but only a few of them think that a clock (a robust one ...) tics the same rythm if transported or not. That is in full contradiction with the principles of SR , which specifie :

« The laws by which the states of physical systems undergo changes are not affected , whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other. »

The simplest example one can find is the following : take a train in uniform movement , a small pendulum swinging on it. It is well-known that its rythm is independant of its translatory movement.

Why then is it engraved in gold letters : clocks that are moving with respect to an inertial system of observation are measured to be running more slowly (than the clocks attached to that system) ? Literally that seems true. But as we shall see it is also the source of many misunderstandings , or even erreneous interpretations. More , it mask's , *bêtement* , a deeper knowledge of the Minkowski space.

A bit of theory.

The Minkowski space is a pseudo-euclidian vector space. Its simplified plane representation is of course falsely euclidian. Take a look at the following figures :

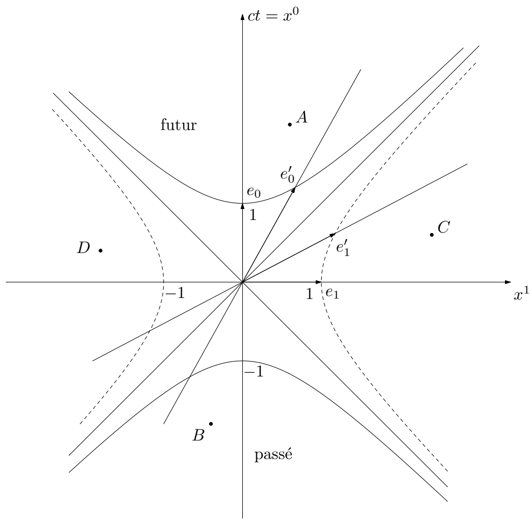

Figure 1.

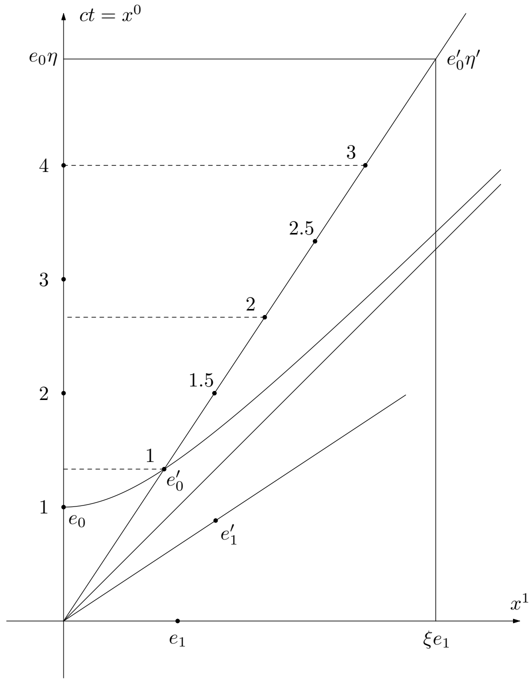

Figure 2.

In the first, two reference frames $(e_0\ ,\ e_1)$, $(e'_0\ ,\ e'_1)$ are represented. Note that both are orthonormal ! And :

(1) $\qquad e_0^2 = {e'}_0^2 = 1 \qquad e_1^2 = {e'}_1^2 = -1 \qquad e_0.e_1 = e'_0.e'_1 = 0$

We draw also the two hyperbolas which define the unite vectors (time and space). Their apparent euclidian equations are :

(2) $\qquad y^2 - x^2 = 1 \qquad x^2 - y^2 = 1$

The diagonals are the isotropic directions (whith light velocity $c = 1$).

The second represents the example that we want to expose.

The reference frame $(e_0\,,e_1)$ is supposed to be motionless. $(e'_0\,,e'_1)$ is supposed to be in motion to the right. That is we follow a particle flying between 0 and $(e'_0\,\eta')$ with the velocity (euclidian) $v < 1$. (e'_0, e'_1) is its « proper » reference frame.

If we apply the relations (1) we obtain immediately the following parametrisation :

(3) $\qquad e'_0 = \operatorname{ch}\theta\, e_0 + \operatorname{sh}\theta\, e_1 \qquad e'_1 = \operatorname{sh}\theta\, e_0 + \operatorname{ch}\theta\, e_1 \qquad v = \operatorname{th}\theta = \beta \qquad \gamma = \operatorname{ch}\theta$

(4) $\qquad e'_0 = \gamma e_0 + \gamma\beta e_1 \qquad e'_1 = \gamma\beta e_0 + \gamma e_1 \qquad ({e'}_0^2) = \gamma^2(1 - \beta^2) = 1$

(5) $\qquad \eta' = \gamma(\eta - \beta\xi) \qquad \xi' = \gamma(-\beta\eta + \xi) \qquad \gamma = (1 - v^2)^{-1/2} \geqslant 1$

Those constitude the (simplified) Lorentz transformations. If we apply it to the flying particle we find :

(6) $\qquad \xi = \beta\eta \qquad \eta = \gamma\eta'$

That is the expected result $\eta' = \gamma^{-1}\eta$. The time (called proper time) necessary for the particle (the astronaut ...) to reach its destination , is lesser than the time noted by the observer in its (fixed) frame.

The interpretation is very simple now. We see on figure 2 that there is a sort of « trading »between time and space , when switching between the two frames. That is specialy obvious if we consider the vectorial equation :

(7) $$e_0'\,\eta' = (\gamma e_0 + \gamma\beta e_1)\gamma^{-1}\eta$$

That means , the clock $e_0' = (\gamma e_0 + \gamma\beta e_1)$ gives the time $\eta' = \gamma^{-1}\eta$!

Of course the clock is unitary. Its rythm has not changed ; the clock has fewer minutes to measure !

To resume , we measure time , a scalar quantity. But the clocks are vectorial ! When I tried to introduce that idea in Wikipedia it provoqued a furious rejection. For me it was very natural , I work with Geometric Algebra ; I will come to that later ...

But we have a test to pass. Time measured by the astronaut should be independent of the observer's frame. Now take a look at figure 3 :

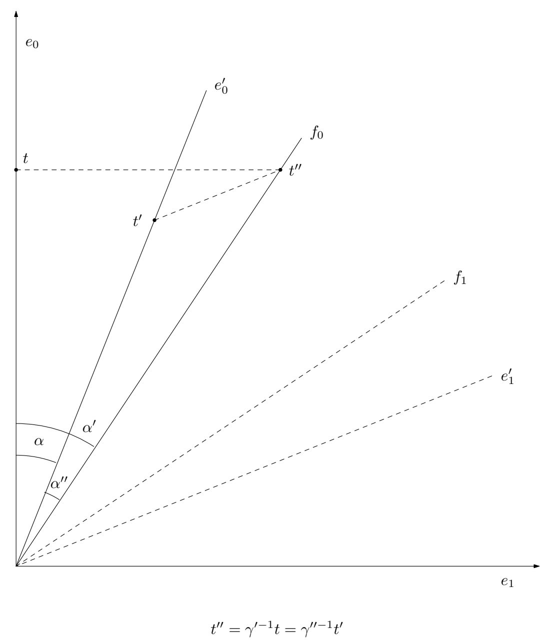

Figure 3.

We will explicate later the symbols $\alpha, \alpha', \alpha''$, for the moment we look at the relationship between t and t'', between t' and t'' . No calculation is necessary ; you have just to replace (e_0, e_1) by (e_0', e_1') and you find obviously (because the two frames are orthogonal) :

(8) $$t'' = \gamma'^{-1}t = \gamma''^{-1}t' \qquad \gamma' = \left(1 - v'^2\right)^{-1/2} \qquad \gamma'' = \left(1 - v''^2\right)^{-1/2}$$

where v' and v'' are the velocities (of the flying object) measured in the two frames.

That means there is no difference of nature between the two frames. The fact we choose one of them looking truely orthogonal has no significance. And that means overall : t'' is independent !

Here we can stress the fact that the orientation of the unit vector f_0 is the key factor for the determination of flying time. That is an intrinsic propriety of the flat spacetime , which is completely left aside by the traditional approach.

Some pratical cases.

Perhaps one of the much publizised case of time dilatation is the muon experiment. We follow the example exposed in http://hyperphysics.phy-astr.gsu.edu . The objective is to demonstrate that only relativity is able explain the number of muons reaching the sea-level.

We are told that the muon , from 10000 m height sees (!) a distance of only 2000 m . But the observer thinks that the internal clock of the muon ticks slow.

Obviously both of these affirmations are false. The muon sees nothing and the internal clock is insensible to the velocity. Its internal clock measures $t' = \gamma^{-1} t$ (see figure 2) and $t' = \lambda\tau$ where τ represents halve-livetime of the muon[1]. Why disguise the real facts ?

Now let us take a look at twin paradox (Wikipedia). We take for granted the formulas employed to describe the whole journey. We have to compare the following elements :

(9) $$\Delta t' = 2T_c\left(1 - \frac{V^2}{c^2}\right)^{1/2} + 4\frac{c}{a}\,\mathrm{arsinh}\left(\frac{aT_a}{c}\right)$$

(10) $$\Delta t = 2T_c + 4T_a$$

$4T_a$ is the time where constant acceleration or deceleration ($=g$) is applied to the rocket ; $2T_c$ is the time where the rocket is floating free at velocity V . T_a , T_c , Δt , are measured with a clock in a fixed reference frame. $\Delta t'$ is the total time registered by the flying clock. T_c was chosen freely. For example :

(11) $$T_c = 10\,\mathrm{years} \qquad a = g = 9,81 m/s^2 \qquad V = 0,8c \qquad c = 2,998\ 10^8 m/s$$

(12) $$aT_a = V\left(1 - \frac{V^2}{c^2}\right)^{-1/2} \qquad V = aT_a\left(1 + \frac{a^2T_a^2}{c^2}\right)^{-1/2}$$

We obtain :

(13) $$aT_a = 1,33c \qquad T_a = 3,123\ 10^7 s \qquad \frac{c}{a}\,\mathrm{arsinh}\left(\frac{aT_a}{c}\right) = 3,357\ 10^7 s$$

(14) $$\Delta t' = 12,0\,\mathrm{years} + 4,25\,\mathrm{years} = 16,25\,\mathrm{years}$$

(15) $$\Delta t = 20,0\,\mathrm{years} + 4,0\,\mathrm{years} = 24,0\,\mathrm{years}$$

We note that the incidence of the variable periods on $(\Delta t - \Delta t')$ is very small (3%). Almost all the shrinking of time is due to the free floating spacetime route. That is the result , not of clocks ticking slow , but of trading between space and time.

We can renew the Hafele-Keating experiment. Take a jet-plane let it do a round trip on the Earth , 40000km at 600km/h. We neglect the fact it is not straight line and that the Earth is not a inertial frame (thus we find a medium time between two opposite flights). We find :

(16) $$T_a = 17,0s \qquad aT_a/c = 5,56\ 10^{-7} \qquad \frac{c}{a}\,\mathrm{arsinh}\left(\frac{aT_a}{c}\right) \simeq T_a(1 - 5,15 10^{-14})$$

(17) $$T_c = 240012s \qquad T_c\left(1 - \frac{V^2}{c^2}\right)^{1/2} = T_c(1 - 1,05 10^{-13})$$

(18) $$\Delta t - \Delta t' = 240012 \text{ x } 1{,}05\ 10^{-13} + 17 \text{ x } 5{,}15\ 10^{-14} \simeq 25\ 10^{-9} s$$

The clock has not changed its rythm , it has simply fewer nanoseconds to measure !

1. One must oberve that the elementary particle muon cannot possess an internal clock ! But it has follow the laws of the weak interaction , which are of course independent of velo

Addendum in .... geometric algebra !

Geometric Algebra (GA) should be the preferred mathematical instrument to study relativity. Its vectorial approach is ideal to manipulate the hyperbolic geometry which constitudes the core of the spacetime algebra.

Start again with figures 1 and 2. Call α the parameters. We get :

(19) $e'_0 = \gamma e_0 + \gamma\beta e_1 = \exp^{\alpha e_1 e_0} e_0$ $\qquad e'_1 = \gamma\beta e_0 + \gamma e_1 = \exp^{\alpha e_1 e_0} e_1$

and more generally :

(20) $e'_\mu = R e_\mu \tilde{R}$ $\qquad R = \exp^{\alpha e_1 e_0 /2}$

with :

(21) $\exp^{\alpha e_1 e_0} = \mathrm{ch}\alpha + e_1 e_0\, \mathrm{sh}\alpha$

That means we can introduce an « angle » α with :

(22) $\beta = v = \mathrm{th}\alpha$ $\qquad \gamma = \mathrm{ch}\alpha = (1 - v^2)^{-1/2}$ $\qquad \beta\gamma = \mathrm{sh}\alpha$

We recall :

(23) $e_0.e'_0 = e_0.[(\gamma + \gamma\beta e_1 e_0) e_0] = \gamma$

Thus (figure 2) :

(24) $e_0 .(\eta' e'_0) = \gamma\eta'$ $\qquad \eta' = \gamma^{-1}\eta$

We define (figure 3) the « angle » α' :

(25) $f_0 = \exp^{\alpha' e_1 e_0} e_0$ with $\mathrm{ch}(\alpha') = \gamma'$ $\quad \mathrm{sh}(\alpha') = \beta'\gamma'$ $\quad \beta' = w$

As we see , in the falsely euclidian figure , e_0 is transformed in e'_0 by the hyperbolic « rotation » α , and in f_0 by the « rotation » α' . That justifies our graphisme. Of course in numerical traduction α and α' are not angles !

Now we can find an elegant way to describe a change of reference frames. We introduce (figure 3) the new frame (e'_0 , e'_0) with the « angle »α , and then call α'' the « angle » between e'_0 and f_0 . We call u the velocity f_0 relative to (e'_0 , e'_1) . That is :

(26) $f_0 = \exp^{\alpha'' e'_1 e'_0} e'_0$ with $\mathrm{ch}(\alpha'') = \gamma''$ $\quad \mathrm{sh}(\alpha'') = \beta''\gamma''$ $\quad \beta'' = u$

By (19) we have :

(27) $e_0 = \exp^{-\alpha e_1 e_0} e'_0$ $\qquad e_1 = \exp^{-\alpha e_1 e_0} e'_1$

Thus by (25) :

(28) $f_0 = \exp^{(\alpha' - \alpha) e_1 e_0} e'_0$

We know :

(29) $e_1 e_0 = e'_1 e'_0$!!

Thus :

(30) $f_0 = \exp^{(\alpha' - \alpha) e'_1 e'_0} e'_0$ $\qquad \mathrm{ch}(\alpha' - \alpha) = \gamma''$ $\qquad \mathrm{sh}(\alpha' - \alpha) = \gamma''\beta''$ $\qquad \beta'' = u$

and :

(31) $\alpha'' = \alpha' - \alpha$!!

The α parameters are additive.

Georges Ringeisen

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